

A Probabilistic Determination of Fuzzy System with Evaluation

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Abstract: The term “fuzzy logic” is rather ambiguous because it refers to problems and method that belongs to different field of investigation. it is possible to find three meaning for the expression “fuzzy logic”. In its most popular acceptance, it refers to numerical computations based on fuzzy rules, for the purpose of modeling a control law in system engineering. Interestingly, the original motivation of fuzzy logic control was to represent expert knowledge in a rule based style and to build a standard control law that start in artificial intelligence.

Keywords: Fuzzy Set; Fuzzy logic; Fuzzy inference system; neural network; Fuzzy rules; Neuro fuzzy system.

I. INTRODUCTION

A fuzzy system is an alternative to traditional notions of set membership and logic that has its origins in ancient Greek philosophy, and applications at the leading edge of Artificial Intelligence. Yet, despite its long-standing origins, it is a relatively new field, and as such leaves much room for development. This paper will present the foundations of fuzzy systems, along with some of the more noteworthy objections to its use, with examples drawn from current research in the field of Artificial Intelligence. Ultimately, it will be demonstrated that the use of fuzzy systems makes a viable addition to the field of Artificial Intelligence, and perhaps more generally to formal mathematics as a whole.

II. HISTORY: BUILT OF FUZZY LOGIC

The precision of mathematics owes its success in large part to the efforts of Aristotle and the philosophers who preceded him. It was Plato who laid the foundation for what would become fuzzy logic, indicating that there was a third region (beyond True and False) where these opposites "tumbled about." Other, more modern philosophers echoed his sentiments, notably Hegel, Marx, and Engels. The idea of grade of membership, which is the concept that became the backbone of fuzzy set theory, occurred to him in 1964, which lead to the publication of his seminal paper on fuzzy sets in 1965 and the birth of fuzzy logic technology.

III. CONCEPT OF FUZZY LOGIC

The step in establishing a complete system of fuzzy logic is to define the operations of EMPTY, EQUAL, COMPLEMENT

(NOT), CONTAINMENT, UNION (OR), and INTERSECTION (AND). Before we can do this rigorously, we must state some formal definitions:

Definition 1: Let X be some set of objects, with elements noted as x . Thus,

$$X = \{x\}.$$

Definition 2: A fuzzy set A in X is characterized by a membership function $\mu_A(x)$ which maps each point in X onto the real interval $[0.0, 1.0]$. As $\mu_A(x)$ approaches 1.0, the "grade of membership" of x in A increases.

Definition 3: A is EMPTY if for all x , $\mu_A(x) = 0.0$. Definition 4: $A = B$ if for all x : $\mu_A(x) = \mu_B(x)$ [or, $\mu_A = \mu_B$].

Definition 5: $\mu_{A'} = 1 - \mu_A$.

Definition 6: A is CONTAINED in B if $\mu_A \leq \mu_B$.

Definition 7: $C = A \text{ UNION } B$, where: $\mu_C(x) = \text{MAX}(\mu_A(x), \mu_B(x))$. Definition 8: $C = A \text{ INTERSECTION } B$ where: $\mu_C(x) = \text{MIN}(\mu_A(x), \mu_B(x))$.

It is important to note the last two operations, UNION (OR) and INTERSECTION (AND), which represent the clearest point of departure from a probabilistic theory for sets to fuzzy sets. Operationally, the differences are as follows:

For independent events, the probabilistic operation for AND is multiplication, which (it can be argued) is counterintuitive for fuzzy systems. For example, let us presume that $x = \text{Bob}$, S is the fuzzy set of smart people, and T is the fuzzy set of tall people. Then, if $\mu_S(x) = 0.90$ and $\mu_T(x) = 0.90$, the probabilistic result would be:

$$\mu_S(x) * \mu_T(x) = 0.81$$

where as the fuzzy result would be:

$$\text{MIN}(\mu_S(x), \mu_T(x)) = 0.90$$

The probabilistic calculation yields a result that is lower than either of the two initial values, which when viewed as "the chance of knowing" makes good sense.

IV. FUZZY LOGIC, SETS, PROBABILITY

The term **fuzzy logic** is used in two senses:

Narrow sense: Fuzzy logic is a branch of fuzzy set theory, which deals (as logical systems do) with the representation and inference from knowledge. Fuzzy logic, unlike other logical

systems, deals with imprecise or uncertain knowledge. In this narrow, and perhaps correct sense, fuzzy logic is just one of the branches of fuzzy set theory.

Broad Sense: fuzzy logic synonymously with fuzzy set theory. Theory of fuzzy sets and fuzzy logic has been applied to problems in a variety of fields: taxonomy; topology; linguistics; logic; automata theory; game theory; pattern recognition; medicine; law; decision support; Information retrieval; etc.

And more recently fuzzy machines have been developed including: automatic train control; tunnel digging machinery; washing machines; rice cookers; vacuum cleaners; air conditioners, etc.

Fuzzy logic is a set of mathematical principles for knowledge representation based on degrees of membership.

Unlike two-valued Boolean logic, fuzzy logic is multi-valued. It deals with degrees of membership and degrees of truth.

Fuzzy logic uses the continuum of logical values between 0 (completely false) and 1 (completely true). Instead of just black and white, it employs the spectrum of colours, accepting that things can be partly true and partly false at the same time.

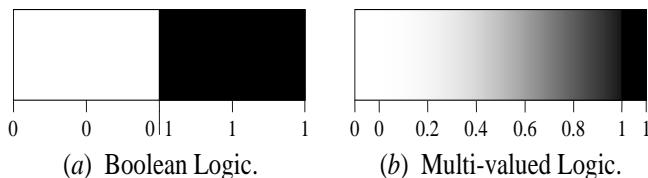


Figure 1. Fuzzy sets

However, our own language is also the supreme expression of sets. For example, car indicates the set of cars. When we say a car, we mean one out of the set of cars. The classical example in fuzzy sets is tall men. The elements of the fuzzy set depend on their height.

Let X be the universe of discourse and its elements be denoted as x . In the classical set theory, crisp set A of X is defined as function $f_A(x)$ called the characteristic function of A :

$$f_A(x) : X \rightarrow \{0, 1\}, \text{ where}$$

$$f_A(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A \end{cases}$$

This set maps universe X to a set of two elements. For any element x of universe X of set A , and is equal to 0 if x is not an element of A . , characteristic function $f_A(x)$ is equal to 1 if x is an element .

In the fuzzy theory, fuzzy set A of universe X is defined by function $\mu_A(x)$ called the membership function of set A .

$$\mu_A(x) : X \rightarrow \{0, 1\},$$

$$\text{where } \mu_A(x) = 1 \text{ if } x \text{ is totally in } A;$$

$$\mu_A(x) = 0 \text{ if } x \text{ is not in } A$$

$$0 < \mu_A(x) < 1 \text{ if } x \text{ is partly in } A.$$

This set allows a continuum of possible choices. For any element x of universe X , membership function $\mu_A(x)$ equals the degree to which x is an element of set A . This degree, a value between 0 and 1, represents the **degree of membership**, also called **membership value**, of element x in set A .

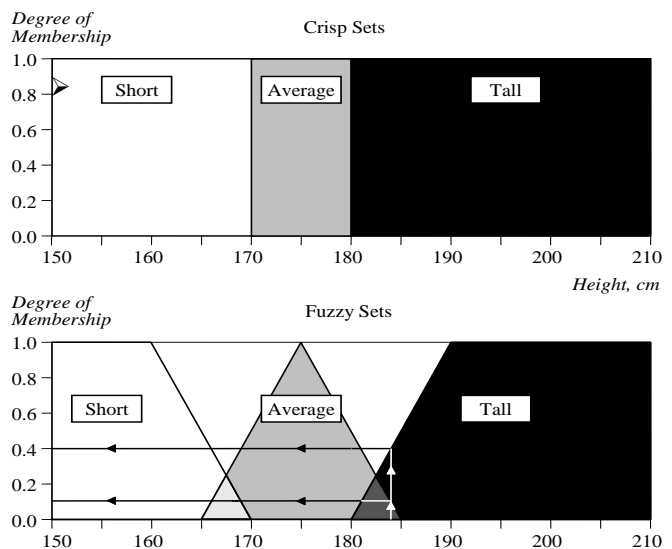


Figure 2. Probability

In the probabilistic approach, a logic sentence is labeled with a real number in the range 0 to 1, 0 meaning the sentence is completely false, and 1 meaning it is completely true. A label of 0.5 Means there is equal chance that the sentence is true or false.

A more general statement of Bayes theorem covers the case where we have some evidence E , and a range of hypotheses H_1, H_2, \dots, H_n which could be drawn from it.

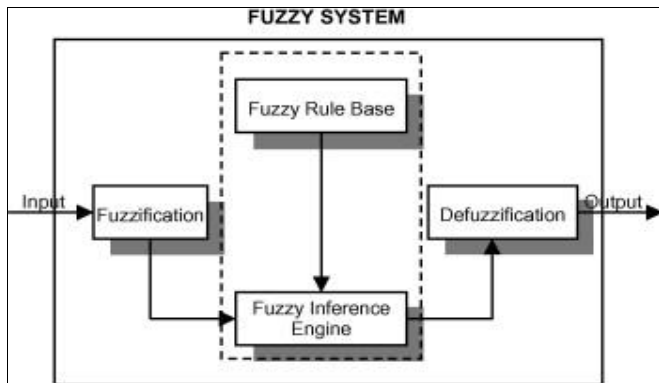
The probability of any particular hypotheses is:

$$\Pr(H_i|E) = \frac{\Pr(E|H_i) \cdot \Pr(H_i)}{\sum_{k=1}^n \Pr(E|H_k) \cdot \Pr(H_k)}$$

The problem with this is it makes the assumption that the statistical data on the relationships of the evidence with the hypotheses are known, and that all relationships between evidence and hypotheses $\Pr(E|H_k)$ are independent of each other. In general this indicates that as the number of factors we are considering increases it becomes increasingly difficult to be able to account for the possible interferences between them.

V. ROLE OF FUZZINESS IN ARTIFICIAL INTELLIGENCE

The collection of these fuzzy rules forms the rule base for the fuzzy logic system. Using suitable inference procedure, the truth value for the antecedent of each rule is computed, and applied to the consequent part of each rule. This results in one fuzzy subset to be assigned to each output variable for each rule. Again, by using suitable composition procedure, all the fuzzy subsets assigned to each output variable are combined together to form a single fuzzy subset for each output variable. Finally, defuzzification is applied to convert the fuzzy output set to a crisp output.



VI. CONCLUSION

It should be noted that Fuzzy Logic is only a small part of the logics available to science. In artificial Intelligence for instance, many researchers find modal and Temporal logics much more relevant to their work. Fuzzy Logic is not the only way of processing information which is uncertain. Probability theory, including Bayes Theorem, provides additional and effective ways of processing uncertain information for many types of application. Fuzzy system, including Fuzzy logic and fuzzy set theory provide a rich and meaningful addition to standard logic.

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